SDSU MASTERS of HOMELAND SECURITY

GEOL600 SENSOR NETWORKS

NETWORK THEORY

- Graph theory
- Seven bridges of Konigsberg
- Four color theorem
- Snarks
- Drawing graphs
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- Network theory
- Complex networks
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- AdHoc Routing Protocols
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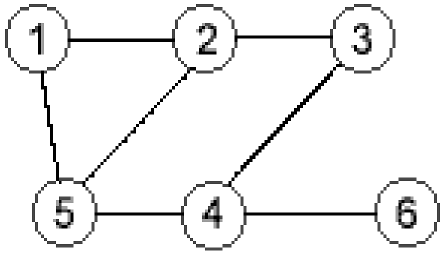
GRAPH THEORY



A graph is a set of objects called **vertices** (or Nodes) connected by links called **edges** (or Arcs) which can be directed. If the graph is directed the direction is indicated by drawing an arrow.

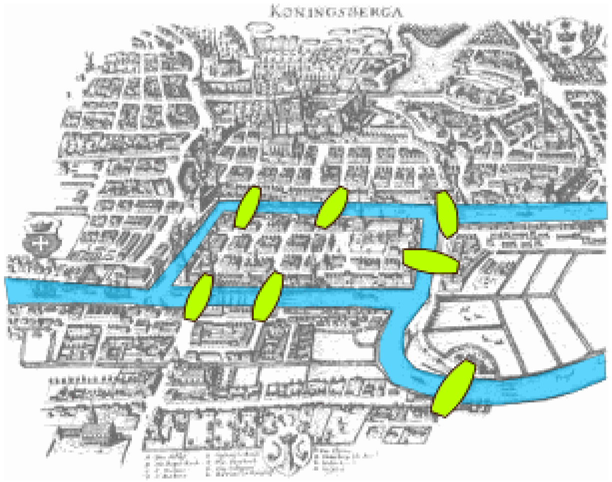
Many structures can be represented as graphs and many problems of practical interest can be represented by graphs.
The link structure of a website can be represented by a directed graph: the vertices are the webpages and there is a directed edge from page A to page B if and only if A contains a link to B.

Development of algorithms to handle graphs is of major interest in computer science.

A graph structure can be extended by assigning a weight to each edge, or by making the edges to the graph directional (A links to B, but B does not necessarily link to A, as in webpages), technically called a digraph.
A digraph with weighted edges is called a **network**.

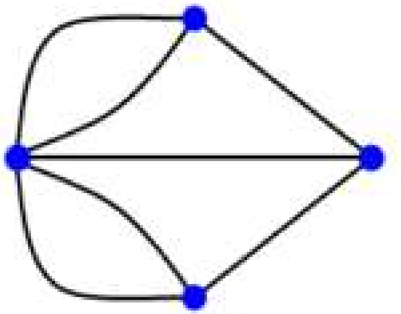
Networks have many uses in the practical side of graph theory, network analysis (for example, to model and analyse traffic networks or to discover the shape of the internet). However, it should be noted that within network analysis, the definition of the term "network" may differ, and may often refer to a simple graph

HISTORY Seven Bridges of Königsberg (1736)



Königsberg, (Prussia) on the river Pregel included two large islands which were connected to each other and the mainland by seven bridges.

Is it possible to walk with a route that crosses each bridge exactly once, and return to the starting point ?



In 1736, Euler proved it is not possible, by formulating the problem in terms of graph theory: replacing each landmass by a node and each bridge with an edge.

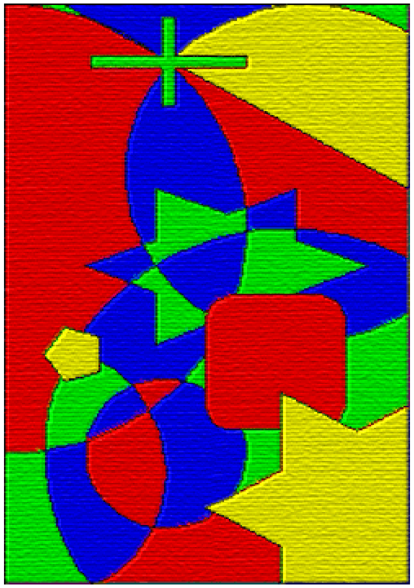
Euler showed that a circuit of the desired form is possible if and only if there are no nodes that have an odd number of edges touching them. Such a walk is called an Eulerian circuit or an Euler tour. Since the graph corresponding to Königsberg has four such nodes, the path is impossible.

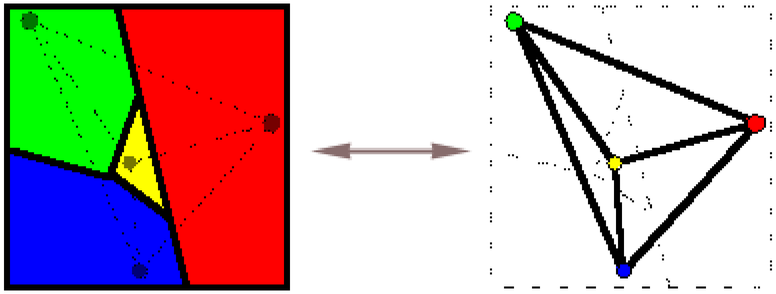
If the starting point does not need to coincide with the end point there can be either zero or two nodes that have an odd number of edges touching them. Such a walk is called an Eulerian trail or Euler walk.

This too is impossible for the seven bridges of Königsberg.

HISTORY: Four color problem (1852 ; 1976)

Four color theorem, first proposed by Guthrie in 1852, states that every possible geographical map can be colored using no more than four colors in such a way that no two adjacent regions are the same color.





To formally state the theorem, it is easiest to rephrase it in graph theory.
It then states that the vertices of every planar graph can be colored with at most four colors so that no two adjacent vertices receive the same color. Or "every planar graph is four-colorable" for short.
Here, every region of the map is replaced by a vertex of the graph, and two vertices are connected by an edge if and only if the two regions share a border segment.

Theorem was proved by Kenneth Appel and Wolfgang Haken in 1976.
The proof of the Four Color Theorem is not simple; it involved lengthy computer checking of more than 100,000 particular cases. Some mathematicians consider this unacceptable, as the proof cannot be reviewed;

The proof does not provide any insight as to why the conjecture is true.
The theorem is true, but unexplained.

Appel K & Haken W, (Oct 1977) Solution of the Four Color Map Problem,
Scientific American, 237 (4): .108-121.

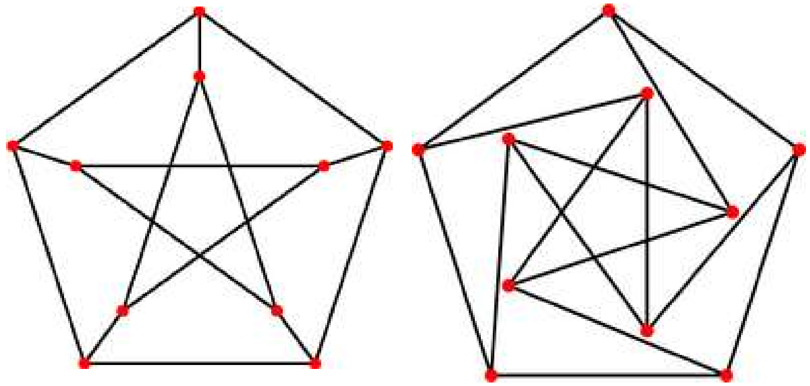
SNARKS

In graph theory, a snark is a connected, bridgeless cubic graph with chromatic index equal to four.

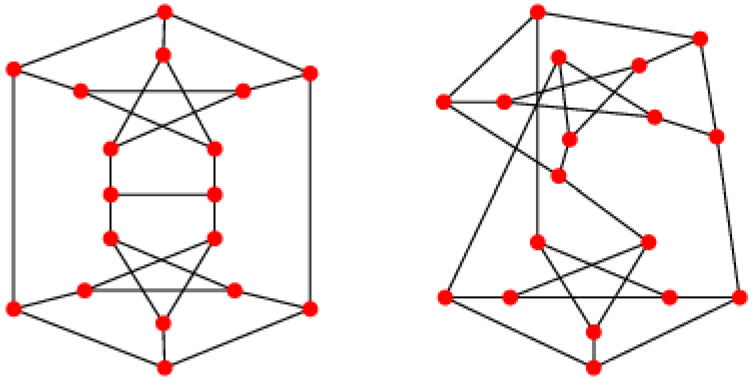
In other words, it is a graph in which every node has three branches, and the edges cannot be colored in fewer than four colors without two edges of the same color meeting at a point.

The four color theorem is equivalent to the statement that no snark is planar.

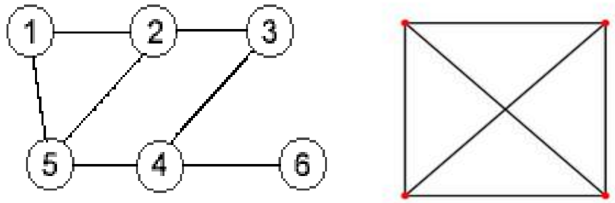
Petersen graph: (smallest snark, 1898)



Blanusa snarks (1946)



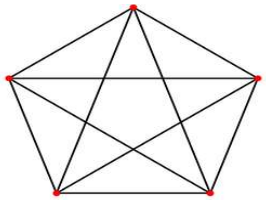
A **planar graph** can be embedded in a plane so that no edges intersect.



Planar

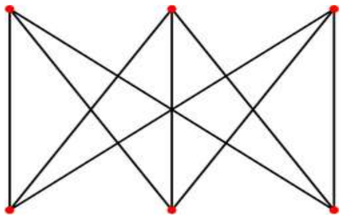
K_5

Complete graph



$K_{3,3}$

Complete bipartite graph



Non Planar

DRAWING GRAPHS

Graphs are represented graphically by drawing a dot for every vertex, and drawing an arc between two vertices if they are connected by an edge. If the graph is directed the direction is indicated by drawing an arrow.

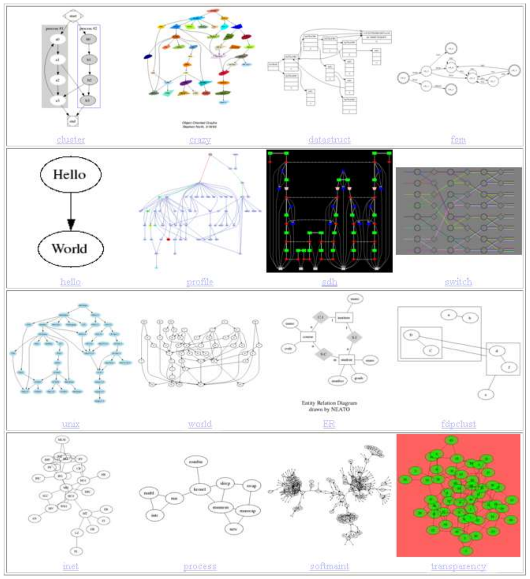
A graph drawing should not be confused with the graph itself (the abstract, non-graphical structure) as there are several ways to structure the graph drawing. All that matters is which vertices are connected to which others by how many edges and not the exact layout.
In practise it is often difficult to decide if two drawings represent the same graph. Depending on the problem domain some layouts may be better suited and easier to understand than others.

Kruja E et al (2001), Short note on history of graph drawing,
Proc.Graph Drawing 2265: 272-286 www.merl.com/papers/TR2001-49/

GraphViz: open source graph visualization software
The Graphviz layout programs take descriptions of graphs in a simple text language, and make diagrams in several useful formats. (supports GXL)

In practice, graphs are usually generated from external data sources, but they can also be created and edited manually, either as raw text files or within a graphical editor. (Graphviz is not a Visio replacement)

www.graphviz.org/



GRAPH DATA STRUCTURES

The data structure used to store graphs depends on the graph structure and the algorithm used for manipulating them.
Theoretically distinguishable list and matrix structures but in reality the best structure is often a combination of both. Lists are preferred for sparse graphs whilst. Matrices provide faster access but consume memory if graph is large.

LIST STRUCTURES

Incidence list - The edges are represented by an array containing pairs (ordered if directed) of vertices (that the edge connects) and eventually weight and other data.

Adjacency list - Each node has a list of which nodes it is adjacent to. This can sometimes result in "overkill" in an undirected graph as vertex 3 may be in the list for node 2, then node 2 must be in the list for node 3. This representation is easier to find all the nodes which are connected to a single node, since these are explicitly listed.

MATRIX STRUCTURES

Incidence matrix - The graph is represented by a matrix of E (edges) by V (vertices), where [edge, vertex] contains the edge's data (simplest case: 1 - connected, 0 - not connected).

Adjacency matrix - there is an N by N matrix, where N is the number of vertices in the graph. If there is an edge from vertex x to vertex y, then the element $M_{x,y}$ is 1, otherwise it is 0. Easier to find subgraphs, and to reverse graphs if needed.

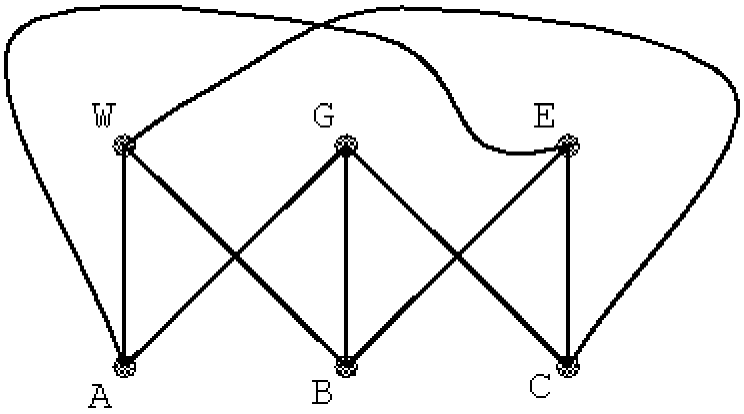
Admittance matrix - is defined as degree matrix minus adjacency matrix and thus contains adjacency information and degree information about the vertices

SUBGRAPH ISOMORPHISM PROBLEM

A common problem is finding subgraphs in a given graph.
As many graph properties are hereditary, if a subgraph has a property then so does the whole graph, for instance, if a graph contains $K_{3,3}$ (the complete bipartite graph) then it is non-planar.

Kuratowski's theorem:
A finite graph is planar if and only if
it does not contain a subgraph that is an expansion of K_5 or $K_{3,3}$.
it does not contain a subgraph that is homemorphic to K_5 or $K_{3,3}$
it does not have K_5 or $K_{3,3}$ as a minor

K3,3 illustrates the 'three cottage problem',:
There are 3 cottages on a plane (or sphere), each needs to be connected to the gas, electricity and water. Is there any way to do so without any of the lines crossing ?



Finding maximal subgraphs of a certain kind is often a NP-complete problem.
(Computational complexity theory NP complexity class problems can be solved by a Non-deterministic machine in polynomial time; the computation time of a problem where the time *m(n)* is no greater than the a polynomial function of the problem size *n.)*

SINGLE SOURCE SHORTEST PATH PROBLEM

the single-source shortest path problem is the problem of finding a path between two vertices such that the sum of the weights of its constituent edges is minimized.
A solution to the shortest path problem is sometimes called a pathing algorithm.

The most important algorithms for solving this problem are:

Dijkstra's algorithm — solves single source problem if all edge weights are greater than or equal to zero. Without worsening the run time, this algorithm can in fact compute the shortest paths from a given start point s to all other nodes.

Bellman-Ford algorithm — solves single source problem if edge weights may be negative.

A* pathing algorithm — a heuristic for single source shortest paths.

Floyd-Warshall algorithm — solves all pairs shortest paths.

Johnson's algorithm — solves all pairs shortest paths, may be faster than Floyd-Warshall on sparse graphs.

A related problem is the traveling salesman problem, which is the problem of finding the shortest path that goes through every node exactly once, and returns to the start. That problem is NP-hard, so an efficient solution is not likely to exist.

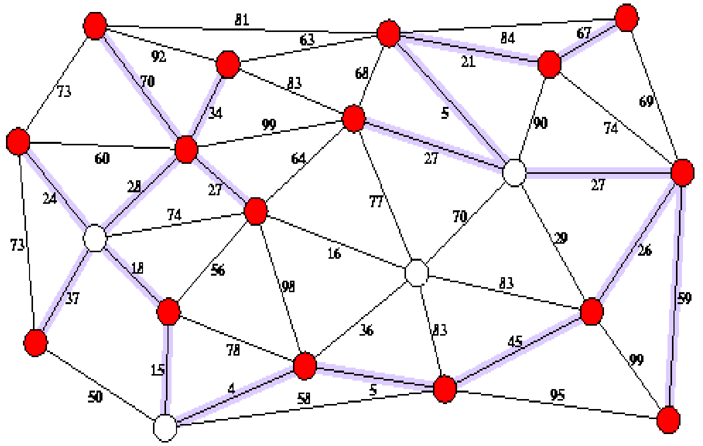
In computer networking / telecommunications, this shortest path problem is sometimes called the min-delay path problem and usually tied with a widest path problem.

STEINER TREE PROBLEM

The Steiner tree problem is a combinatorial optimization problem
In its most general setting it is stated in a way similar to that of the minimum spanning tree problem: given a set V of points (vertices), it is required to interconnect them by a network (graph) of shortest length provided that it is allowed to add new vertices to the network.
The latter possibility is the difference from the minimum spanning tree problem.

These new vertices introduced to decrease the total length of connection are known as Steiner points or Steiner vertices.
It is proven that the resulting connection is a tree, known as the Steiner tree.
There may be several Steiner trees for a given set of vertices.

The Steiner tree problem has applications in circuit layout or network design. Most versions of the Steiner tree problem are NP-complete, i.e., computationally hard. Some restricted cases can be solved in polynomial time. In practice, heuristics are used.



One common approximation to the Euclidean Steiner tree problem is to compute the Euclidean minimum spanning tree.

NETWORK THEORY

Network theory is another branch of applied mathematics with the same general subject matter as graph theory, namely a graph is a representation of a symmetric relation, and a directed graph for a general binary relation.
The approach is application centered, in particular to computer networks but not limited to those. Network theory is also used to describe the use of social network maps within the social sciences.

COMPLEX NETWORKS

Complex networks are the backbone of complex systems.
They are special networks at the edge of chaos where the degree of connectivity is neither regular nor random.

The most complex networks of the real world are either **small-world networks** or **scale-free networks** at the border between regular and random networks. They can be described by means of mathematics and Graph Theory.

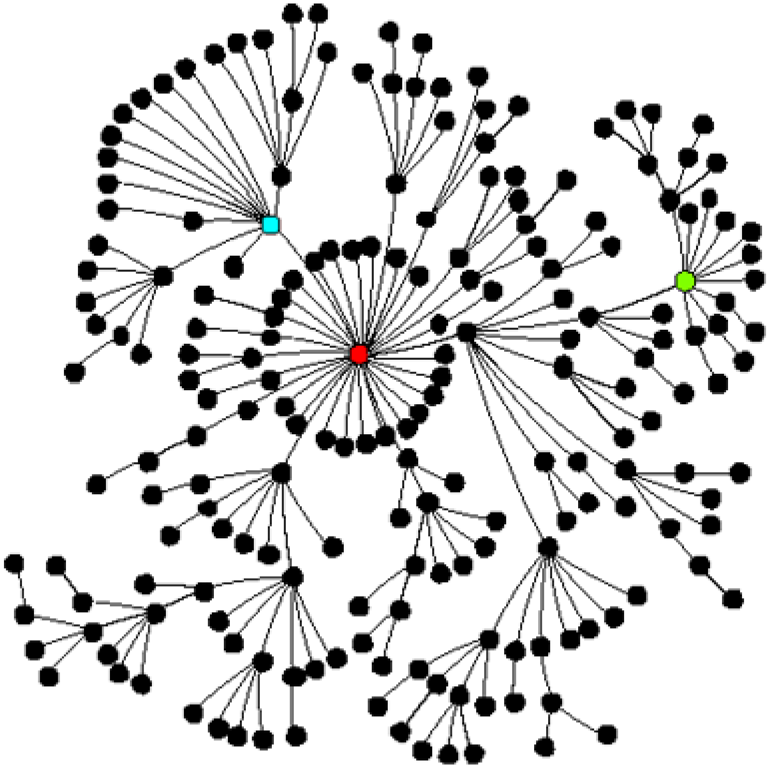
Both classes of complex networks, small-world and scale-free networks are very similar. Small-world networks or graphs emerge through the random rewiring of regular grids or lattices: adding randomness to order.
Scale-free networks arise in networks if you add order to randomnes: instead of considering a pure random growth of a network, you consider a random growth with preferential attachment.

The small-world property can be associated with global connectivity and the shortest path length, it arises in regular networks through the addition of random shortcuts. The scale-free property can be associated with local connectivity, it arises in random networks through clustering.

SCALE FREE (ARISTOCRATIC) NETWORKS

In a scale-free network the distribution of connectivity is extremely uneven. Some nodes act as "very connected" hubs using a power-law distribution $y = ax^k$

This kind of connectedness dramatically influences the way the network operates, including how it responds to catastrophic events.
The Internet, World Wide Web and many other large-scale networks have been shown to be scale-free networks.



The term "scale-free" was coined by Barabasi et al in 1998, they mapped the connectedness of the World Wide Web and found that the web does not have an even distribution of connectivity (so-called "random connectivity").

Instead, a very few network nodes (called "hubs") are far more connected than other nodes. In general, they found that the probability P(k) that a node in the network connects with k other nodes was, in a given network, proportional to $k^{-\gamma}$.

There is a simple explanation for this behavior.

Many networks expand through the addition of nodes to an existing network, and those nodes attach preferentially to nodes already well-connected.

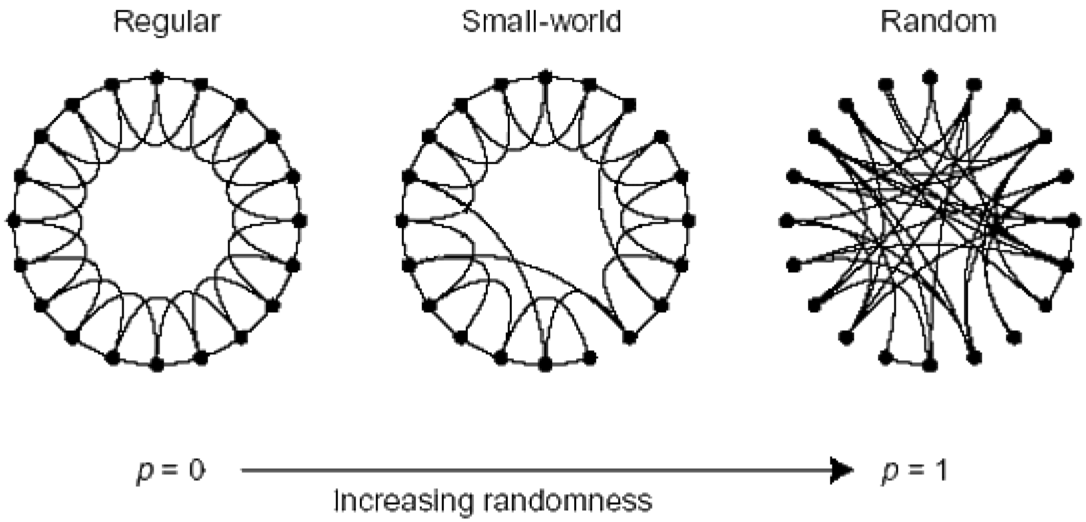
It has been found that many networks (including those describing interrelationships of objects) are scale-free. They have been identified in the dispersal of STDs, air travel connections, and many kinds of computer networks.

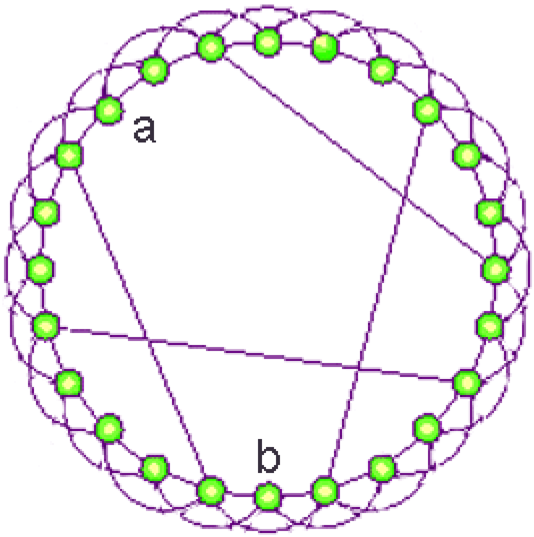
Many have studied collaboration networks, in which nodes represent people, and the links between nodes represent some kind of collaboration between them. Many of these have also been found to be scale-free networks.

Computer networks that are also scale-free networks are significantly different from random connectivity networks in the presence of failure. If nodes fail randomly, scale-free networks behave even better than random connectivity networks, because random failures are unlikely to harm an important hub. However, if the failure of nodes is not random, scale-free networks can fail catastrophically. For example, an intelligent attacker can essentially destroy an entire scale-free network by intelligently identifying and attacking its key hubs.

Thus, the realization that certain networks are scale-free is important to security.

SMALL WORLD NETWORKS





A small-world network is a specific kind of network (to be more precise a special kind of a complex network) in which the distribution of connectivity is not confined to a certain scale, and where every node can be reached from every other by a small number of hops or steps. It is a generalisation of the small-world phenomenon

The small world hypothesis, tested experimentally , is the idea that two arbitrary people are connected by only six degrees of separation, i.e. the diameter of the corresponding graph of social connections is not much larger than six. http://smallworld.columbia.edu/results.html

The small-world phenomenon applies to social networks. Duncan J. Watts and Steven Strogatz (1998) have identified it as a general feature of certain networks and propose that a similar phenomenon can occur in any network.

They propose that we can measure whether a network is a small world or not according to two graph measurements of the network: clustering coefficient and mean-shortest path length.

If the clustering coefficient is significantly higher than would be expected for a random network, and the mean shortest-path length is lower than would be expected for a regular network, then the network is a small world.

It can be seen how this works for the small-world phenomenon: most people have a relatively small circle of friends who generally all know each other (highly clustered), but the shortest-path length from one person to any other is possibly very short.

Examples:
University of Virginia : Oracle of Bacon, Star Links

www.cs.virginia.edu/oracle/

www.cs.virginia.edu/oracle/star_links.html

www.canyouhearmeyet.com/

WIRELESS MESH NETWORKS

Mesh networking is a way to route data between nodes. It allows for continuous connections and reconfiguration around blocked paths by "hopping" from node to node until a connection can be established.

Mesh networks are self-healing: the network can still operate even when a node breaks down or a connection goes bad.

As a result, a very reliable network is formed.

Networking infrastructure is decentralised and inexpensive, as each node need only transmit as far as the next node. Nodes act as repeaters to transmit data from nearby nodes to peers that are too far away to reach, resulting in a network that can span a large distance, especially over rough or difficult terrain.

Mesh networks are also extremely reliable, as each node is connected to several other nodes. If one node drops out of the network, due to hardware failure or any other reason, its neighbours simply find another route. Extra capacity can be installed by simply adding more nodes. Mesh networks may involve either fixed or mobile devices.

The principle is similar to the way packets travel around the Internet - data will hop from one device to another until it reaches a given destination. Dynamic routing capabilities included in each device allow this to happen. To implement such dynamic routing capabilities, each device needs to communicate its routing information to every device it connects with, "almost in real time". Each device then determines what to do with the data it receives - either pass it on to the next device or keep it. The routing algorithm used should attempt to always ensure that the data takes the most appropriate (fastest) route to its destination.

The choice of radio technology for wireless mesh networks is crucial.
In Infrastructure mode the more laptops connect the less bandwidth is available for each user.
With mesh technology and adaptive radio, devices in a mesh network will only connect with other devices that are in a set range. Like a natural load balancing system the more devices the more bandwidth available, provided that the number of hops in the average communications path is kept low.

To prevent increased hop count from cancelling out the advantages of multiple transceivers, one common type of architecture for a mobile mesh network includes multiple fixed base stations with high-bandwidth terrestrial links that will provide gateways to services, the Internet and other fixed base stations. The "cut through" bandwidth of the base station infrastructure must be substantial for the network to operate effectively.

ADHOC ROUTING PROTOCOLS

There are a large number of competing schemes for routing packets across mesh networks. They can be classified as proactive (table driven), reactive (on demand), hierarchical, geographical, power aware, multicast or geocast.
Some of these are

TORA (Temporally-Ordered Routing Algorithm)
OORP (OrderOne Routing Protocol)
AODV (Ad-hoc On Demand Distance Vector)
OLSR (Optimized Link State Routing protocol)
HSLS (Hazy-Sighted Link State)

Refer to en.wikipedia.org/wiki/Ad_hoc_protocol_list for comprehensive listing

MANET (Mobile Ad-Hoc Network)

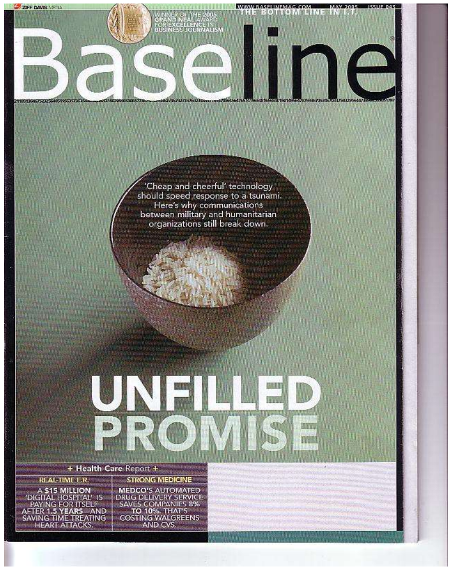
A MANET is a self-configuring network of mobile routers (and associated hosts) connected by wireless links—the union of which form an arbitrary topology. The routers are free to move randomly and organise themselves arbitrarily; thus, the network's wireless topology may change rapidly and unpredictably. Such a network may operate in a standalone fashion, or may be connected to the larger Internet.

The earliest MANETs were called "packet radio" networks, and were sponsored by DARPA in the early 1970s. These early packet radio systems predated the Internet, and indeed were part of the motivation of the original Internet Protocol suite. Later DARPA experiments included the Survivable Radio Network (SURAN) project, which took place in the 1980s. Another third wave of academic activity started in the mid 1990s with the advent of inexpensive 802.11 radio cards for personal computers. Current MANETs are designed primarly for military utility; examples include JTRS and NTDR.

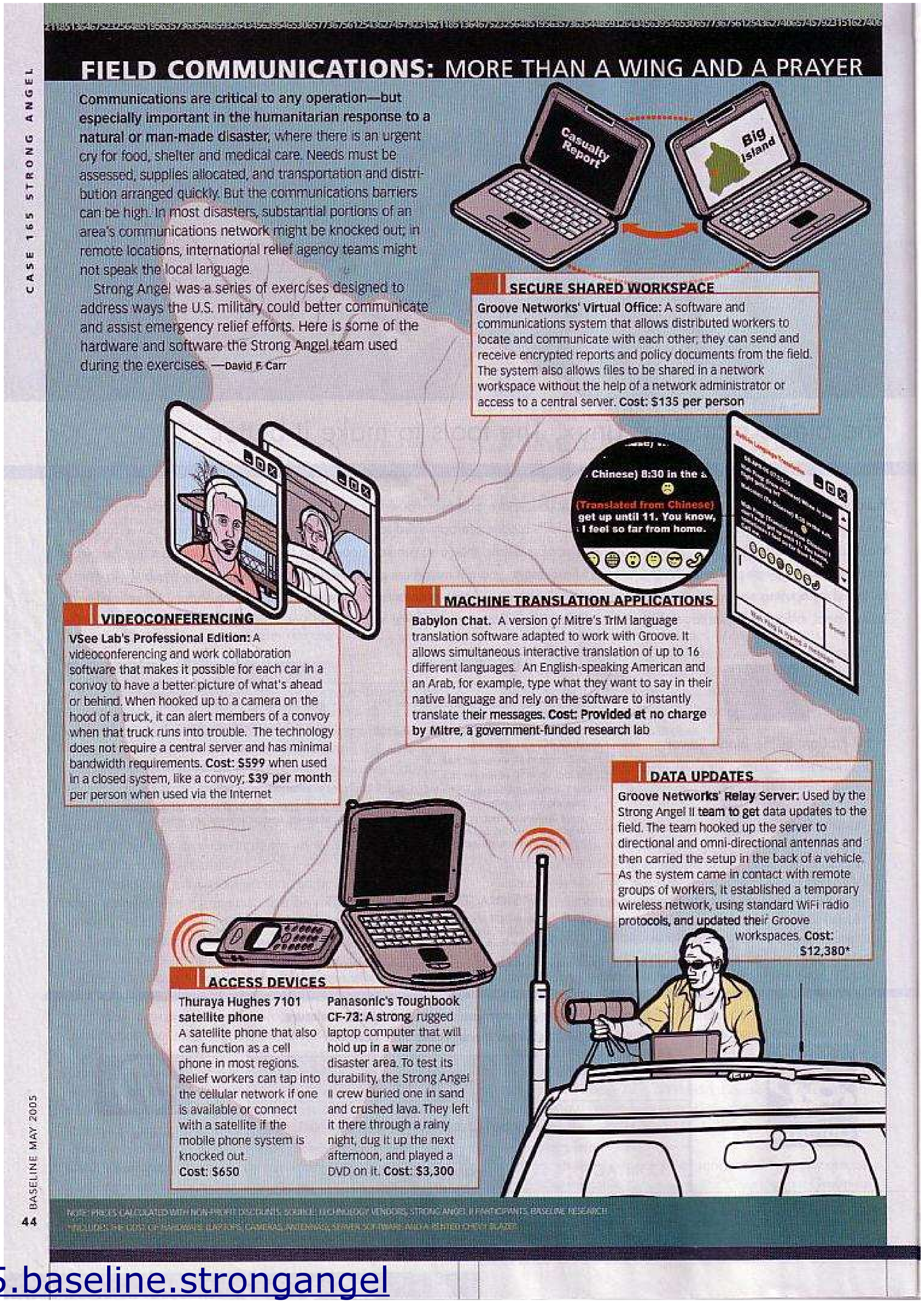
The popular IEEE 802.11 ("Wi-Fi") wireless protocol incorporates an ad-hoc networking system when no wireless access points are present, although it would be considered a very low-grade ad-hoc protocol by specialists in the field. The IEEE 802.11 system only handles traffic within a local "cloud" of wireless devices. Each node transmits and receives data, but does not route anything between the network's systems.
However, higher-level protocols can be used to aggregate various IEEE 802.11 ad-hoc networks into MANETs.

RANT

PONY EXPRESS



Baseline article:
Carr D (2005)
“Unfilled Promise”



FIELD COMMUNICATIONS: MORE THAN A WING AND A PRAYER

Communications are critical to any operation—but especially important in the humanitarian response to a natural or man-made disaster, where there is an urgent cry for food, shelter and medical care. Needs must be assessed, supplies allocated, and transportation and distribution arranged quickly. But the communications barriers can be high. In most disasters, substantial portions of an area's communications network might be knocked out; in remote locations, international relief agency teams might not speak the local language.

Strong Angel was a series of exercises designed to address ways the U.S. military could better communicate and assist emergency relief efforts. Here is some of the hardware and software the Strong Angel team used during the exercises. —David F. Carr

SECURE SHARED WORKSPACE

Groove Networks' Virtual Office: A software and communications system that allows distributed workers to locate and communicate with each other; they can send and receive encrypted reports and policy documents from the field. The system also allows files to be shared in a network workspace without the help of a network administrator or access to a central server. **Cost: $135 per person**

VIDEOCONFERENCING

VSee Lab's Professional Edition: A videoconferencing and work collaboration software that makes it possible for each car in a convoy to have a better picture of what's ahead or behind. When hooked up to a camera on the hood of a truck, it can alert members of a convoy when that truck runs into trouble. The technology does not require a central server and has minimal bandwidth requirements. **Cost: $599** when used in a closed system, like a convoy; **$39 per month** per person when used via the Internet

MACHINE TRANSLATION APPLICATIONS

Babylon Chat. A version of Mitre's TrIM language translation software adapted to work with Groove. It allows simultaneous interactive translation of up to 16 different languages. An English-speaking American and an Arab, for example, type what they want to say in their native language and rely on the software to instantly translate their messages. **Cost: Provided at no charge by Mitre, a government-funded research lab**

DATA UPDATES

Groove Networks' Relay Server: Used by the Strong Angel II team to get data updates to the field. The team hooked up the server to directional and omni-directional antennas and then carried the setup in the back of a vehicle. As the system came in contact with remote groups of workers, it established a temporary wireless network, using standard WiFi radio protocols, and updated their Groove workspaces. **Cost: $12,380***

ACCESS DEVICES

Thuraya Hughes 7101 satellite phone
A satellite phone that also can function as a cell phone in most regions. Relief workers can tap into the cellular network if one is available or connect with a satellite if the mobile phone system is knocked out.
Cost: $650

Panasonic's Toughbook CF-73: A strong, rugged laptop computer that will hold up in a war zone or disaster area. To test its durability, the Strong Angel II crew buried one in sand and crushed lava. They left it there through a rainy night, dug it up the next afternoon, and played a DVD on it. **Cost: $3,300**

NOTE: PRICES CALCULATED WITH NON-PROFIT DISCOUNTS. SOURCE: TECHNOLOGY VENDORS, STRONG ANGEL II PARTICIPANTS, BASELINE RESEARCH
*INCLUDES THE COST OF HARDWARE (LAPTOPS, CAMERAS, ANTENNAS), SERVER SOFTWARE AND A RENTED CHEVY BLAZER

projects.mindtel.com/2005/05.baseline.strongangel